

11-4 Geometric Series

sum of a geometric sequence

$$2 + 6 + 18 + 54 + 162$$

$$Sum = 242$$

$$2 + 6 + 18 + 54 + 162$$

$$r = 3$$

$$S_5 = 2 + 6 + 18 + 54 + 162$$

$$-3 \cdot S_5 = -6 - 18 - 54 - 162 - 486$$

$$-2 \cdot S_5 = 2 - 486$$

$$S_5 = \frac{2 - 486}{-2} = 242$$

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

$$a_n = a_1 r^{n-1}$$

$$S_n = \frac{a_1 - r a_n}{1 - r}$$

$$= \frac{a_1 - r a_1 r^{n-1}}{1 - r}$$

$$= \frac{a_1 - a_1 r^n}{1 - r} = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_n = \frac{a_1 - r a_n}{1 - r}$$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

$$r \neq 1$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Ex 1:
Find the sum.
 $a_1 = 7776$
 $a_n = 6$

$$r = \frac{-1}{6}$$

$$S_n = \frac{a_1 - r a_n}{1 - r}$$

$$\frac{777}{\frac{7}{6}} = \frac{7776 - (\frac{1}{6})(6)}{1 - (\frac{1}{6})}$$

$$6666$$

Ex 2:

$$a_1 = ?$$

$$S_8 = 765$$

$$r = 2$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$\frac{765}{1} = \frac{a_1(1 - 2^8)}{1 - 2}$$

$$-765 = a_1(1 - 2^8)$$

$$3 = a_1$$

Sigma Notation

$$\sum_{n=1}^{12} 3 \cdot 2^{n-1}$$

Expanded

$$= 3 + 6 + 12 + \dots$$

Find the sum $a_n = 3 \cdot 2^{n-1}$

$$S_{12} = \frac{3 - 2(6144)}{1-2} = 12285$$

Sigma Notation

$$\sum_{n=1}^5 6 \cdot \left(\frac{1}{3}\right)^{n-1}$$

Put the following series into sigma notation.

$$48 + 24 + 12 + 6 + \dots + \frac{3}{8}$$

$$a_n = 48 \left(\frac{1}{2}\right)^{n-1}$$

$$\sum_{n=1}^8 48 \left(\frac{1}{2}\right)^{n-1}$$

Find the sum
 $95 \frac{3}{8}$

$$\frac{3}{8} = 48 \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{128} = \frac{1}{2}^{n-1}$$

$$\log_{\frac{1}{2}} \frac{1}{128} = (n-1) \log_{\frac{1}{2}} \frac{1}{2}$$

$$7 = n-1$$

$$8 = n$$

29. **HEALTH** Contagious diseases can spread very quickly. Suppose five people are ill during the first week of an epidemic and that each person who is ill spreads the disease to four people by the end of the next week. By the end of the tenth week of the epidemic, how many people have been affected by the illness?

$$a_1 = 5$$

$$r = 4$$

$$S_{10} = \frac{5(1-4^{10})}{1-4} = 1,747,625$$

HW

p597

15-21odd, 27, 30, 35, 41, 43